SUMMER SCHOOL ON FLUIDS AND TURBULENCE
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Amphithéatre Jordan, Bât. Braconnier Université Lyon 1

Lecture series

Anne-Laure Dalibard. (Sorbonne Université). *"Boundary layers in fluid dynamics"*

In these lectures, I will review some methods and results on the construction of fluid boundary layers. In the first lecture, I will focus on the linear case, and I will present a general method to compute the boundary layer sizes and profiles. I will also briefly address the limitations of this method. The next two lectures will be devoted to the Prandtl equation. I will present its derivation, and will make an overview of the results and of the remaining open problems, distinguishing between the stationary and the time dependent setting.

Thierry Gallay. (Université Grenoble Alpes). *"Vortex Rings in Viscous and Ideal Flows"*

Abstract: Vortex rings are three-dimensional flows characterized by a vorticity distribution that is concentrated in a solid torus. Such flows are most conveniently modeled by axisymmetric solutions without swirl of the 3D Euler or Navier-Stokes equations. In the inviscid case, stationary solutions in a moving frame can be constructed by variational or functional analytic techniques, and persistence of the spatial localization over finite time intervals can also be established for general solutions. In the viscous case, it is most natural to consider the situation where the initial vorticity is a circular vortex filament, the circulation of which may be arbitrarily large compared to viscosity. In that case, one can show that the axisymmetric Navier-Stokes equations are globally well-posed, and it is possible to perform an asymptotic expansion of the solution in the vanishing viscosity limit. In this way, we obtain a rigorous justification of the binormal flow approximation for the advection speed of a viscous vortex ring with Gaussian profile. These lectures are based on joint work with Vladimir Sverak (Minneapolis).

Francisco Gancedo. (Universidad de Sevilla). *"Construction of fluid interfaces for incompressible flows"*

Abstract: In this course we discuss new techniques to construct moving fluid interfaces of incompressible flows. We deal with the Muskat problem, Surface Quasi-Geostrophic (SQG) fronts and Navier-Stokes free boundary. Muskat models the evolution of an incompressible fluid filtered in porous media driven by gravity. We show that initial 2D Lipschitz graphs of arbitrary size provide global-in-time well-posedness for the stable problem. SQG models the dynamics of atmospheric and oceanic flows. Temperature front solutions show numerical evidence of finite-time curvature blow-up. We construct fronts with unbounded curvature providing a local-in-time well-posedness theory. Finally, using inhomogeneous Navier-Stokes equations we study the dynamics of two incompressible immiscible fluids in 2D. We prove that initial small viscosity contrast yields global-in-time regularity.

Invited talks

Alexey Cheskidov. (University of Illinois at Chicago) *"Intermittency in fluid flows"*

Abstract: I will discuss the notion of intermittency in turbulent fluid flows, particularly a volumetric description based on flatness factors that recovers intermittency corrections to the structure exponents in an explicit way. For instance, the predictions of the Frisch-Parisi multifractal formalism can be recovered in a systematic and rigorous way. I will also discuss the use of spacial and temporal intermittency in designing convex integration schemes to construct nonunique solutions to the Navier-Stokes and transport equation.

Diego Córdoba. (ICMAT - Madrid, Spain). *"Non existence and strong ill-posedness in $C^{k,\gamma}$ and Sobolev spaces for the generalized SQG equation"*

Abstract: In this talk we present recent results on the existence of solutions of the generalized Surface Quasi-geostrophic equations (SQG) that initially are in C^k, $C^{k,\gamma}$ or in supercritical Sobolev spaces, but lose that prescribe regularity for t ¿ 0. This is a joint work with Luis Martinez-Zoroa.

Gianluca Crippa. (Universität Basel). *"An elementary proof of existence and uniqueness for the Euler flow in uniformly localized Yudovich spaces"*

Abstract: I will revisit Yudovich's well-posedness result for the 2-dimensional Euler equations. I will derive an explicit modulus of continuity for the velocity, depending on the growth in p of the (uniformly localized) L^p norms of the vorticity. If the growth is moderate at infinity, the modulus of continuity is Osgood and this allows to show uniqueness. I will also show how existence can be proved in (uniformly localized) L^p spaces for the vorticity. All the arguments are fully elementary, make no use of Sobolev spaces, Calderon-Zygmund theory, or Paley-Littlewood decompositions, and provide explicit expressions for all the objects involved. This is a joint work with Giorgio Stefani (SISSA Trieste).

Mimi Dai. (University of Illinois at Chicago). *"Singularity formation for reduced models of fluid equations"*

Abstract: The non-resistive electron magnetohydrodynamics (MHD) has some peculiar connection with the Euler equation. Some one-dimensional reduced models are suggested as an attempt to build intuition toward understanding the nonlinear structure and dynamics of the electron MHD. In particular, finite time singularities may occur for the reduced models in some contexts.

Eduard Feireisl. (Czech Academy of Sciences). Joint work with Peter Bella and Florian Oschmann (TU Dortmund). *"The incompressible limit for the Rayleigh-Benard convection problem"*

Abstract: We consider a general compressible viscous and heat conducting fluid confined between two parallel plates and heated from the bottom. The time evolution of the fluid is described by the Navier-Stokes-Fourier system considered in the regime of low Mach and Froude numbers suitably interrelated. The asymptotic limit is identified as the Oberbeck-Boussinesq system supplemented with non-local boundary conditions for the temperature deviation.

Dragoş Iftimie. (Université Claude Bernard Lyon 1). *"On the incompressible α-Euler equations in the exterior of a vanishing disk"*

Abstract: We consider the α-Euler equations in the exterior of a small fixed disk of radius ϵ. We assume that the initial potential vorticity is compactly supported and independent of ϵ, and that the circulation of the unfiltered velocity on the boundary of the disk does not depend on ϵ. We prove that the solution of this problem converges, as $\epsilon \to 0$, to the solution of a modified α-Euler equation in the full plane where an additional Dirac located at the center of the disk is imposed in the potential vorticity. This is a joint work with V. Busuioc, H. Nussenzveig Lopes and M. Lopes Filho.

Christophe Prange. (CNRS and Cergy Paris Université). *"Concentration and quantitative regularity for the Navier-Stokes equations"*

Abstract: In this talk I will show concentration phenomena near potential singularities of the three-dimensional Navier-Stokes equations. I will also investigate the connection between concentration estimates and quantitative regularity. This is a joint work with Tobias Barker (University of Bath).

Frédéric Rousset. (Université Paris-Saclay). *TBA*

Gregory Seregin. (Université Paris-Saclay). *"Long time behaviour and local regularity for solutions to the Navier-Stokes equations"*

Abstract: We shall discuss the problem of local regularity for solutions to the Navier-Stokes equations under certain scale-invariant conditions. Using zooming and duality, we reduce it to the problem of long time behaviour of a certain Stokes equations with a drift. This is in part a joint work with M. Schonbek.

Franck Sueur. (Université de Bordeaux). *"A few remarks on the transport-Stokes system"*

Abstract: In this talk, I will present a joint work with Amina Mecherbet regarding the so-called transport-Stokes system which describes sedimentation of inertialess suspensions in a viscous flow. This system couples a transport equation and the steady Stokes equations in the full three-dimensional space. Our results deal with global existence and uniqueness result for $L^1 \cap L^p$ initial densities where p greater than or equal to 3. Moreover, we prove that, in the case where $p > 3$, the flow map which describes the trajectories of these solutions is analytic with respect to time. Finally we establish the small-time global exact controllability of the transport-Stokes system. These results extend to the transport-Stokes system some classical results on the incompressible Euler system.

Klaus Widmayer. (Universität Zürich). *"Global axisymmetric Euler flows with rotation"*

Abstract: We discuss the construction of a class of global, dynamical solutions to the 3d Euler equations near the stationary state given by uniform "rigid body" rotation. These solutions are axisymmetric, of Sobolev regularity and have non-vanishing swirl. At the heart of this result is a dispersive effect due to rotation, which is captured in our "method of partial symmetries". This approach is adapted to maximally exploit the symmetries of this anisotropic problem, both for the linear and nonlinear analysis, and allows to globally propagate sharp decay estimates. This is joint work with Y. Guo and B. Pausader (Brown University).

Andrej Zlatoš. (University of California San Diego). *"Euler equations on general planar domains"*

Abstract: Bounded vorticity solutions to the 2D Euler equations on singular domains are typically not close to Lipschitz near boundary singularities, which makes their uniqueness a difficult open problem. I will present a general sufficient condition on the geometry of the domain that guarantees global uniqueness for all solutions initially constant near the boundary. This condition is only slightly more restrictive than exclusion of corners with angles greater than π and, in particular, is satisfied by all convex domains. Its proof is based on showing that fluid particle trajectories for general bounded vorticity solutions cannot reach the boundary in finite time. The condition also turns out to be sharp in the latter sense: there are domains that come arbitrarily close to satisfying it and on which particle trajectories can reach the boundary in finite time. The above results also extend to positive vorticity solutions on singular domains that may even contain corners with angles greater than π.